

Dynamics of Fully Nonlinear Drift Wave-Zonal Flow Turbulence System in Plasmas

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Abstract

We present numerical simulations of fully nonlinear drift wave-zonal flow (DW-ZF) turbulence systems in a nonuniform magnetoplasma. In our model, the drift wave (DW) dynamics is pseudo-three-dimensional (pseudo-3D) and accounts for self-interactions among finite amplitude DWs and their coupling to the two-dimensional (2D) large amplitude zonal flows (ZFs). The dynamics of the 2D ZFs in the presence of the Reynolds stress of the pseudo-3D DWs is governed by the driven Euler equation. Numerical simulations of the fully nonlinear coupled DW-ZF equations reveal that shortscale DW turbulence leads to nonlinear saturated dipolar vortices, whereas the ZF sets in spontaneously and is dominated by a monopolar vortex structure. The ZFs are found to suppress the cross-field turbulent particle transport. The present results provide a better model for understanding the coexistence of short- and large-scale coherent structures, as well as associated subdued cross-field particle transport in magnetically confined fusion plasmas.

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It is widely recognized that the presence of large scale sheared flows [1, 2, 3] [also referred to as convective cells (CCs) or zonal flows (ZFs)] is detrimental to regulating the cross-field turbulent transport in magnetically confined fusion plasmas. The ZF is characterized by poloidally and toroidally symmetric structure with radial variation, and the relative zonal flow potential fluctuation (in comparison with T_e/e , where T_e is the electron temperature and e is the magnitude of the electron charge) is much smaller than the relative zonal flow density perturbation (in comparison with the equilibrium plasma number density n_0). The large scale Zonal jets also occur in various planetary atmosphere, where they are nonlinearly generated by the Rossby waves [4, 5], and influence the atmospheric wind circulation [6, 7].

In magnetically confined fusion plasmas, there exist free energy sources in the form of density, temperature, and magnetic field inhomogeneities, which are responsible for exciting the low-frequency (in comparison with the ion gyrofrequency), short scale (of the order of the ion gyroradius or the ion sound gyroradius) DW-like fluctuations [8, 9, 10]. The linearly growing drift modes interact among themselves and attain large amplitudes in due course of time. The Reynolds stress of finite amplitude DWs, in turn, nonlinearly generate convective cells (CCs) and sheared flows/ZFs [11, 12, 13, 14, 15, 16, 17], via three-wave decay and modulational instabilities [12], respectively. There are recent review articles presenting the status of theoretical and simulation works [17], as well as experimental observations [18] concerning the dynamics of DW-ZF turbulence system. Specifically, some numerical simulations [17] lend support to the experimental observation that the DW turbulence and transport levels are reduced in the presence of the sheared flows/ZFs.

Recently, Guo *et al.* [19] used the governing equations of Ref. [12] for the DW-CC turbulence system to investigate the radial spreading of the DW-ZF turbulence via soliton formation. However, the authors of Ref. [19] completely neglected self-interactions among drift waves and zonal flows, which are very important in the study of nonlinearly coupled finite amplitude drift and zonal flow disturbances in nonuniform magnetoplasmas.

In this Letter, we present simulation results of fully nonlinear DW-ZF turbulence systems, which exhibit the coexistence of drift dipolar vortices and a radially symmetric monopolar zonal flow vortex. The effect of the latter on the cross-field turbulent transport is examined. Our investigation is based on the governing equations for the DW-ZF turbulence systems that incorporate the Hasegawa-Mima (HM) self-interaction nonlinearity [20] in the nonlinear dynamics of the DW which are nonlinearly exciting CCs/ZFs. Furthermore, we also account

for nonlinear interactions among the CCs/ZFs and obtain the driven Euler equation for the dynamics of finite amplitude CCs/ZFs. The generalization of the governing equations for fully nonlinear DW-ZF turbulence systems is rather essential for the investigation of the formation of coherent nonlinear structures that control the transport properties and confinement of tokamak plasmas.

We consider a nonuniform magnetoplasma in an external magnetic field $\hat{\mathbf{z}}B_0$, where $\hat{\mathbf{z}}$ is the unit vector along the z -axis in a Cartesian coordinate system and B_0 is the strength of the homogeneous magnetic field. The density gradient $\partial n_0/\partial x$ is along the x -axis. In the presence of the finite amplitude low-frequency (in comparison with the ion gyrofrequency $\omega_{ci} = eB_0/m_i c$, where m_i is the ion mass and c is the speed of light in vacuum) electrostatic DWs and ZFs, the perpendicular (to $\hat{\mathbf{z}}$ component of the electron and ion fluid velocities [13] are, respectively,

$$\mathbf{u}_{e\perp}^d \approx \frac{c}{B_0} \hat{\mathbf{z}} \times \nabla \phi - \frac{c}{B_0 n_e} \hat{\mathbf{z}} \times \nabla (n_e T_e) \equiv \mathbf{u}_{EB}^d + \mathbf{u}_{De}^d, \quad (1)$$

$$\mathbf{u}_{e\perp}^z \approx (c/B_0) \hat{\mathbf{z}} \times \nabla \psi \equiv \mathbf{u}_{EB}^z, \quad (2)$$

$$\begin{aligned} \mathbf{u}_{i\perp}^d \approx \mathbf{u}_{EB}^d + \mathbf{u}_{Di}^d - \frac{c}{B_0 \omega_{ci}} \left(\frac{\partial}{\partial t} + \nu_{in} - 0.3 \nu_{ii} \rho_i^2 \nabla_{\perp}^2 + \mathbf{u}_{EB}^d \cdot \nabla + \mathbf{u}_{Di}^d \cdot \nabla \right) \nabla_{\perp} \phi \\ - \frac{c}{B_0 \omega_{ci}} \left[(\mathbf{u}_{EB}^d \cdot \nabla) \nabla_{\perp} \psi + (\mathbf{u}_{EB}^z \cdot \nabla) \nabla_{\perp} \phi \right], \end{aligned} \quad (3)$$

and

$$\mathbf{u}_{i\perp}^z \approx \mathbf{u}_{EB}^z - \frac{c}{B_0 \omega_{ci}} \left[\left(\frac{\partial}{\partial t} + \nu_{in} - 0.3 \nu_{ii} \rho_i^2 \nabla_{\perp}^2 \right) \nabla_{\perp} \psi + \langle (\mathbf{u}_{EB}^d \cdot \nabla) \nabla_{\perp} \phi \rangle \right], \quad (4)$$

where the superscripts d and z represents quantities associated with the DWs and ZFs, respectively, ϕ and ψ are the electrostatic potentials of the DWs and ZFs, respectively, n_e and n_i are the electron and ion number densities, respectively, $\mathbf{u}_{Di}^d = (c/eB_0 n_i) \hat{\mathbf{z}} \times \nabla (n_i T_i)$ is the ion diamagnetic drift velocity, T_i is the ion temperature, ν_{in} (ν_{ii}) is the ion-neutral (ion-ion) collision frequency, $\rho_i = V_{Ti}/\omega_{ci}$ is the ion gyro-thermal radius, and V_{Ti} is the ion thermal speed, We stress that the self-interaction nonlinearities of the DWs and ZFs are retained in the fluid velocities (3) and (4), respectively. The angular brackets denote averaging over one period of the DWs.

Assuming that $\left| (\partial/\partial t) + \mathbf{u}_{EB}^d \cdot \nabla \right| \ll \nu_{en}$, where ν_{en} is the electron-neutral collision frequency, we obtain from the parallel (to $\hat{\mathbf{z}}$ component) of the electron momentum equation the

magnetic field-aligned electron fluid velocity $u_{ez}^d \approx (1/m_e \nu_{en}) \partial (e\phi - T_e n_{e1}^d / n_0) / \partial z$, where $n_{e1} = (n_e - n_0) \ll n_0$. We can now insert u_{ez}^d into the electron continuity equation to obtain

$$\left[\frac{\partial}{\partial t} + \frac{V_{Te}^2}{\nu_{en}} \frac{\partial^2}{\partial z^2} + (\mathbf{u}_{EB}^d + \mathbf{u}_{EB}^z) \cdot \nabla \right] n_{e1}^d + \mathbf{u}_{EB}^d \cdot \nabla n_0 + \frac{n_0 e}{m_e \nu_{en}} \frac{\partial^2 \phi}{\partial z^2} = 0, \quad (5)$$

where $V_{Te} = (T_e/m_e)^{1/2}$ is the electron thermal speed and m_e is the electron mass. Furthermore, substituting for the ion fluid velocity from (3) into the ion continuity equation we have

$$\begin{aligned} & \left[\frac{\partial}{\partial t} + (\mathbf{u}_{EB}^d + \mathbf{u}_{EB}^z) \cdot \nabla \right] n_{i1}^d + \mathbf{u}_{EB}^d \cdot \nabla (n_0 + n_{i1}^z) \\ & - \frac{n_0 c}{B_0 \omega_{ci}} \left[\left(\frac{\partial}{\partial t} + \nu_{in} - 0.3 \nu_{ii} \rho_i^2 \nabla_{\perp}^2 + \mathbf{u}_{EB}^d \cdot \nabla \right) \nabla_{\perp}^2 \phi + \nabla \cdot (\mathbf{u}_{Di}^d \cdot \nabla) \nabla_{\perp} \phi \right] \\ & - \frac{n_0 c}{B_0 \omega_{ci}} \left[(\mathbf{u}_{EB}^d \cdot \nabla) \nabla_{\perp}^2 \psi + (\mathbf{u}_{EB}^z \cdot \nabla) \nabla_{\perp}^2 \phi \right] = 0, \end{aligned} \quad (6)$$

where the magnetic field-aligned ion dynamics has been ignored, thereby isolating the ion sound waves from our system. The ion density perturbation associated with the ZFs is $n_{i1}^z = (n_0 c / B_0 \omega_{ci}) \nabla_{\perp}^2 \psi$.

Equations (5) and (6), which govern the dynamics of collisional drift waves [21] in the presence of zonal flows, are closed by assuming $n_{e1}^d \approx n_{i1}^d \equiv n_1$, which is a valid approximation in plasmas with $\omega_{pi}^2 \gg \omega_{ci}^2$, where ω_{pi} is the ion plasma frequency. In the linear limit, without the ZFs, Eqs. (5) and (6) yield the DW frequency $\omega_k = -k_y c_s \rho_s / L_n (1 + k_{\perp}^2 \rho_s^2)$ and the growth rate $\gamma_k (\ll \omega_k)$, which are much larger than the damping rate $\nu_{in} + 0.3 \nu_{ii} k_{\perp}^2 \rho_i^2$. The growth rate is $\gamma_k = \nu_{en} \omega_k^2 k_{\perp}^2 / \omega_{LH}^2 k_z^2 (1 + k_{\perp}^2 \rho_s^2)$, where $c_s = (T_e/m_i)^{1/2}$ is the ion sound speed, $\rho_s = c_s / \omega_{ci}$ is the sound gyroradius, $\omega_{LH} = (\omega_{ce} \omega_{ci})^{1/2}$ is the lower-hybrid resonance frequency, $\omega_{ce} = eB_0 / m_e c$ is the electron gyrofrequency, $L_n = (\partial \ln n_0 / \partial x)^{-1}$ is the scale-length of the density gradient, and $\mathbf{k} = \mathbf{k}_{\perp} + \hat{\mathbf{z}} k_z$ is the wave vector.

The equation for the ZFs is obtained by inserting (2) and (4) into the electron and ion continuity equations, and inserting the resultant equations into the Poisson equation, obtaining the driven [by the DW Reynolds stress; the last term in the left-hand side of Eq. (7)] damped (by the ion-neutral collision and ion-gyroviscosity effects) ZF equation

$$\left(\frac{\partial}{\partial t} + \nu_{in} - 0.3 \nu_{ii} \rho_i^2 \nabla_{\perp}^2 + \mathbf{u}_{EB}^z \cdot \nabla \right) \nabla_{\perp}^2 \psi + \langle (\mathbf{u}_{EB}^d \cdot \nabla) \nabla_{\perp}^2 \phi \rangle = 0. \quad (7)$$

For the collisionless DWs, we assume that $|(\partial \phi / \partial t) + (\mathbf{u}_{EB}^d + \mathbf{u}_{EB}^z) \cdot \nabla \phi| \ll (V_{Te}^2 / \nu_{en}) \nabla_{\perp}^2 \phi$ and $\hat{\mathbf{z}} \times \nabla n_0 \cdot \nabla \phi \ll (\omega_{ce} / \nu_{en}) n_0 |\partial^2 \phi / \partial z^2|$, and obtain from (5) the Boltzmann law for the

electron number density perturbation $n_{e1}^2 = n_0 e \phi / T_e$. The latter can be inserted into (6) by assuming that $n_{i1}^d = n_{e1}^d$, so that we have fully nonlinear equation for the DWs in the presence of ZFs

$$\begin{aligned} \frac{\partial \phi}{\partial t} - \frac{c_s \rho_s}{L_n} \frac{\partial \phi}{\partial y} - \rho_s^2 \left[\frac{\partial}{\partial t} + \nu_{in} - 0.3 \nu_{ii} \rho_i^2 \nabla_{\perp}^2 + \frac{c}{B_0} (1 + \sigma) (\hat{\mathbf{z}} \times \nabla \phi) \cdot \nabla \right] \nabla_{\perp}^2 \phi \\ + \frac{c}{B_0} (\hat{\mathbf{z}} \times \nabla \psi) \cdot \nabla (\phi - \rho_s^2 \nabla_{\perp}^2 \phi) = 0, \end{aligned} \quad (8)$$

where $\sigma = T_i / T_e$.

We normalize the time and space variables by ω_{ci-1} and ρ_s , as well as ϕ and ψ by T_e , and the collision frequencies by ω_{ci} . In the normalized units, we can rewrite (7) and (8) as, respectively,

$$\left[\frac{\partial}{\partial t} + \frac{\nu_{in}}{\omega_{ci}} - 0.3 \frac{\nu_{ii}}{\omega_{ci}} \sigma \nabla_{\perp}^2 + (\hat{\mathbf{z}} \times \nabla \psi) \cdot \nabla \right] \nabla_{\perp}^2 \psi + \langle (\hat{\mathbf{z}} \times \nabla \phi \cdot \nabla) \nabla_{\perp}^2 \phi \rangle = 0, \quad (9)$$

and

$$\begin{aligned} \frac{\partial \phi}{\partial t} - \frac{\rho_s}{L_n} \frac{\partial \phi}{\partial y} - \left[\frac{\partial}{\partial t} + \frac{\nu_{in}}{\omega_{ci}} - 0.3 \frac{\nu_{ii}}{\omega_{ci}} \sigma \nabla_{\perp}^2 + (1 + \sigma) (\hat{\mathbf{z}} \times \nabla \phi) \cdot \nabla \right] \nabla_{\perp}^2 \phi \\ + (\hat{\mathbf{z}} \times \nabla \psi) \cdot \nabla (\phi - \nabla_{\perp}^2 \phi) = 0. \end{aligned} \quad (10)$$

We have developed a 2D code to numerically integrate the system of equations (9) and (10), which describe the self-consistent evolution of the DW-ZF turbulence systems. We have chosen $\nu_{in}/\omega_{ci} = 0.1$, $\nu_{ii}/\omega_{ci} = 0.01$, $\sigma = 0.1$, and $\rho_s/L_n = 0.01$. Numerical discretization employs the spatial derivative in Fourier spectral space, while time is discretized using time-split integration algorithm, as prescribed in Ref. [22]. Periodic boundary conditions are used along the x and y directions. A fixed time integration step is used. The conservation of energy [23] is used to check the numerical accuracy and validity of our numerical code during the nonlinear evolution of the small scale drift wave fluctuations and zonal flows. We also make sure that the initial fluctuations are isotropic and do not influence any anisotropic flow during the evolution. Anisotropic flows in the evolution can, however, be generated from a $k_y = 0$ mode that is excited as a result of the nonlinear interactions between the ZFs and small scale DW turbulence. The ZF and DW fields are initialized with a small amplitude and uniform isotropic random spectral distribution of Fourier modes in a 2D computational domain. These fields further evolve through Eqs. (9) and (10) under the influence of nonlinear interactions. Intrinsically, the set of Eqs. (9) and (10) possesses

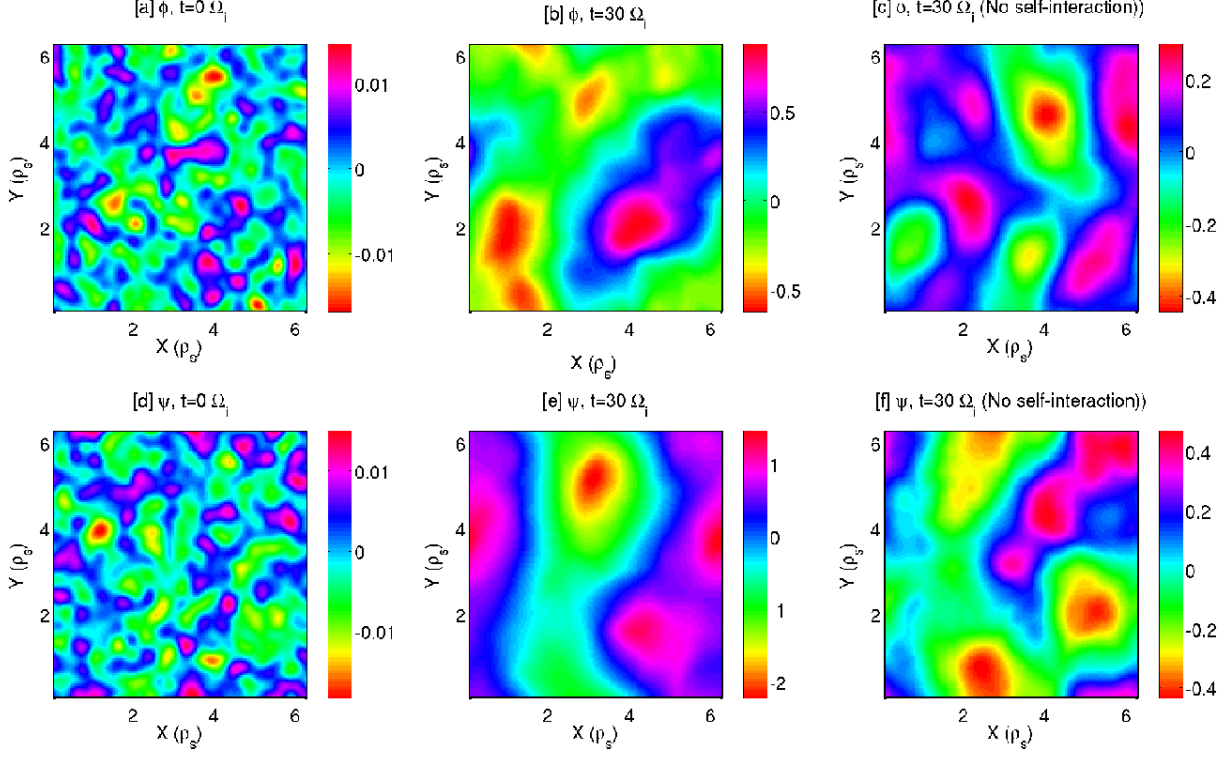


FIG. 1: Evolution of mode structures in our coupled DW-ZF turbulence model from an initial random distribution. In the presence of self-interaction terms, zonal flows are enhanced and quench the DW turbulence more efficiently. Numerical resolution is 256^2 , box size is $2\pi \times 2\pi$.

parametrically unstable modes involving short scale drift waves and zonal flows. In the early phase of simulations, we obtain the growth of small scale DWs. We have carried out two characteristically distinct sets of simulations by switching on and off the self-interaction terms. This enables us to gain considerable insight into the physics of generation of zonal flows and associated transport level in the coupled DW-ZF turbulence systems.

To gain insight into the characteristics nonlinear interactions in our coupled DW-ZF turbulence model, we examine the Hasegawa-Mima-Wakatani (HMW) model [20, 21] that describes the electrostatic drift waves in an inhomogeneous magnetoplasma. First, the ion polarization drift nonlinearity in the HMW model, viz. $\hat{z} \times \nabla \phi \cdot \nabla \nabla^2 \phi$, signifies the self-interaction Reynolds stress that plays a critical role in the formation of the ZFs [12]. This nonlinearity is basically responsible for the generations of the ZFs. Secondly, since in the collisionless DW dynamics, the electron density perturbation follows a Boltzmann law due to the rapid thermalization of electrons along \hat{z} , the nonlinearity $\hat{z} \times \nabla \psi \cdot \nabla \phi$ comes from

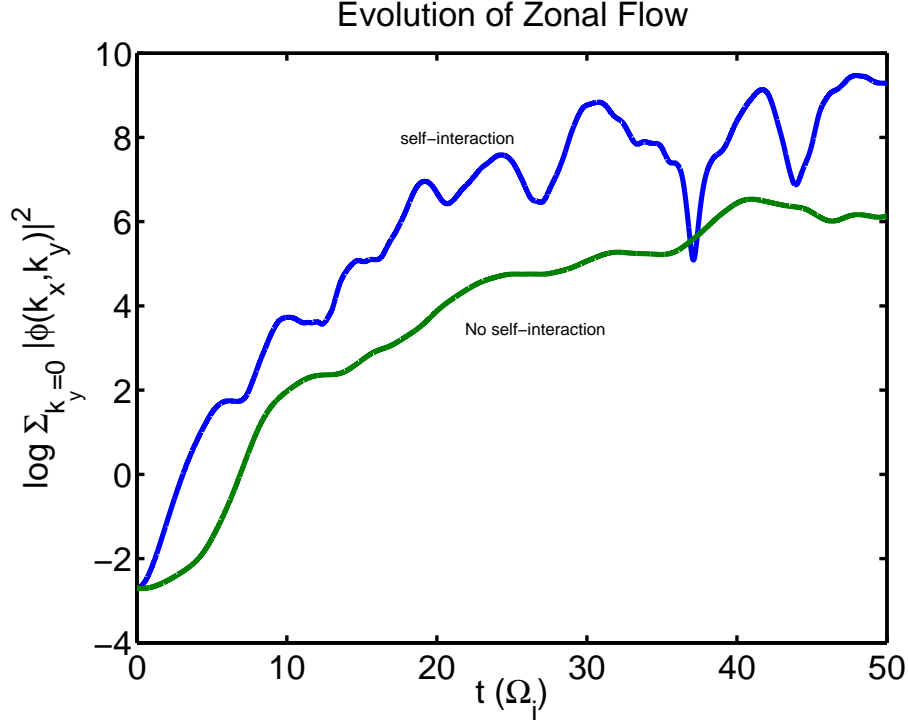


FIG. 2: The self-consistent generation of zonal flows is shown. In the presence of the self-interaction nonlinearity, zonal flows are generated rapidly and their saturated level is also enhanced when compared with the evolution without the self-interaction nonlinearity.

the cross-coupling of the ZF's $\mathbf{E}_{\perp}^z \times \hat{\mathbf{z}}$ particle motion with the drift wave density fluctuation in our model. The role of this nonlinearity has traditionally been identified as a source of suppressing the intensity of the nonlinear flows in the DW turbulence [23]. Nevertheless, the presence of the linear inhomogeneous background can modify the nonlinear mode couplings in a subtle manner. Our objective here is to understand the latter in the context of the coupled DW-ZF turbulence system. The initially isotropic and homogeneous spectral distribution associated with potential fluctuations, as described above, evolve dynamically following the set of Eqs. (9) and (10).

The small amplitude initial drift wave fluctuations are subject to the *modulational instability* on account of their nonlinear coupling with ZFs. The parametrically unstable fluctuations grow rapidly during the early phase of the evolution. The instability eventually saturates via the nonlinear mode couplings in which the DW Reynolds stress, in concert with other nonlinearities in Eqs. (9) and (10), play a critical role. The mode couplings during the nonlinear phase of the evolution leads to the formation of non-symmetric zonal

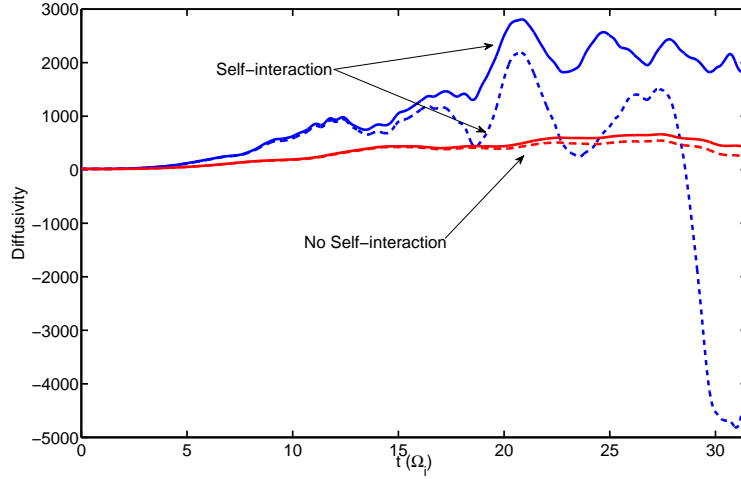


FIG. 3: Evolution of the cross-field diffusivity in the presence (blue curve), as well as in the absence (red) of the self-interaction term. The cross-field diffusivity with and without zonal flows is shown by the dashed and solid curves, respectively. Clearly, the presence of the self-interaction term enhances zonal flows, which dramatically reduce the cross-field diffusivity.

flow structures. This is shown in Fig. 1. Our simulations exhibit that the self-interaction terms not only suppress the modulational instability on a rapid timescales, but they also regulate the generation of the ZFs (see, Figs 1 b, c, e, f). The extent and amplitude of the ZFs in Figs. 1(b) and (e) [with the self-interaction] are larger than that of (c) and (f) [without the self-interaction]. The final (i.e. steady-state) structures, nonetheless, show the formation of a predominantly dipolar vortex in the DW fluctuations, while the ZFs are dominated by a large-scale monopolar-vortex motions. It is noteworthy that the absence of the self-interaction contaminates the flows with more small scale structures (See Figs. 1 c and f).

We next investigate the quantitative evolution of the ZFs, which is depicted in Fig. 2. The spectral transfer of energy in the ZF is estimated from $\sum_k |\phi(k_x, k_y = 0)|^2$. The latter describes a pile up of energy in the $k_y = 0$ mode that is summed up over the entire turbulent spectrum. We find from our simulations that the presence of the self-interaction nonlinearity rapidly suppresses the linear phase of the modulational instability of the DWs. Hence, the linear phase during the evolution terminates rapidly when compared with the no-self-interaction case. Alternatively, the modulational growth rate is enhanced, and it

saturates on a rapid timescales. Consequently, the presence of the self-interactions gives rise to an enhanced level of ZFs, as shown in Fig. 2.

A direct consequence of the enhanced ZFs is to markedly suppress the cross-field turbulent transport, because the sheared flows (in the poloidal direction) associated with the ZFs tear apart the DW fluctuations/eddies, and thereby keeping their amplitudes low. We have computed the cross-field diffusivity in our simulations by using the ion fluxes involving the Boltzmann electron density perturbation and the linear ion polarization drift velocity associated with the nonthermal DWs. The cross-field ion diffusivity reads

$$D = D_B \sum_{\mathbf{k}} \frac{k_y^2 \rho_s^2 |\phi_k|^2}{(1 + k_{\perp}^2 \rho_s^2)}, \quad (11)$$

where $D_B = cT_e/eB_0$ is the Bohm diffusion coefficient and ϕ_k is the spectral potential distribution of the DWs. Note that the ion thermal diffusivity, as defined above, is subdued in the presence of the ZFs, since the latter eventually suppress the DW turbulence so that the steady state cross-field transport level is reduced. Consistent with this scenario, we find, from our simulations, that the onset of the ZFs quenches the cross-field turbulent transport, as shown by the dashed-curve in Fig. 3. Furthermore, due to the vanishing poloidal wavenumber of the ZFs, the sheared flows do not cause any cross-field turbulent transport in magnetized plasmas.

In summary, the most notable point that emerges from our simulations of the coupled DW-ZF turbulence system is the importance and significance of the self-interaction nonlinearity in modeling the low-frequency DW turbulence that is believed to be a critical source for heat and energy losses. A most realistic and accurate understanding of the latter is, therefore, essential for the building and the performance of the next generation controlled thermonuclear fusion reactors, such as the ITER. In the work, we have, for the first time, brought about the importance of self-interaction processes and their role with regard to the cross-field turbulent transport in high-temperature plasmas of thermonuclear fusion devices (e.g. tokamaks). For our purposes, we used a new set of nonlinear equations for the coupled DW-ZF turbulence system, which is a generalization of Ref. 7, by including self-interactions among DWs which drives finite amplitude ZFs. Numerical simulations of the newly obtained nonlinear equations reveal that the coupled DW-ZF turbulence system evolves in the form of short-scale drift dipolar vortices and a large scale monopolar zonal flow structure. The simultaneous presence of the dipolar and monopolar vortices is responsible for a subdued

cross-field turbulent transport in a magnetically confined fusion plasma.

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